

1) Find $f(g(x))$, $g(f(x))$, $f(f(x))$ and $g(g(x))$ and the domain of each.

$$f(x) = x^2$$

$$g(x) = \sqrt{2x - 2}$$

2) Find $f(g(x))$, $g(f(x))$, $f(f(x))$ and $g(g(x))$ and the domain of each.

$$f(x) = \frac{1}{x+2}$$

$$g(x) = \frac{4}{x-1}$$

3) Find $f(g(x))$ for the functions defined as follows:

$$f(x) = \begin{cases} \sqrt{x} & x \geq -1 \\ x^2 - 1 & x < -1 \end{cases} \quad g(x) = \begin{cases} 2x - 1 & x > 0 \\ x^2 & x \leq 0 \end{cases}$$

$$f(x) = \begin{cases} \sqrt{x} & x > 0 \\ \sqrt{x} & -1 \leq x \leq 0 \\ x^2 - 1 & x < -1 \end{cases} \quad f \circ g = \begin{cases} \sqrt{2x-1} & x > 0 \\ \sqrt{x^2} & -1 \leq x \leq 0 \\ (x^2)^2 - 1 & x < -1 \end{cases}$$

$$f \circ g = \begin{cases} \sqrt{2x-1} & x > 0 \\ x & -1 \leq x \leq 0 \\ x^4 - 1 & x < -1 \end{cases}$$

4) Verify algebraically that the following functions are inverses.

$$f(x) = \frac{x-5}{2x+3}$$

$$g(x) = \frac{3x+5}{1-2x}$$